

A NOTE ON UNIQUENESS OF L -FUNCTIONS WITH REGARD TO MULTIPLICITY

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Abstract. By introducing the concept of multiplicity along with weighted sharing of small functions or fixed points, we investigate the uniqueness problems of meromorphic function and a L -function in the extended Selberg class. The results obtained in this paper improves the results due to Liu-Li-Yi ([14]), Sahoo-Halder ([18]), F. Liu-Li-Yi ([15]) and Wen-Jie Hao and Jun-Fan Chen ([3]).

1. Introduction and main results

Selberg studied that the Riemann hypothesis is also true for L -functions in the Selberg class. An L -function based on Riemann Zeta function as the prototype is defined as Dirichlet series

$$L(s) = \sum_{n=1}^{\infty} a(n)n^{-s}, \quad (1.1)$$

of a complex number $s = \sigma + it$ satisfying the following axioms([19]):

- (i) Ramanujan hypothesis: $a(n) \ll n^\epsilon$ for every $\epsilon > 0$;
- (ii) Analytic continuation: There is a non-negative integer m such that $(s - 1)^m L(s)$ is an entire function of finite order;
- (iii) Functional equation: L satisfies a functional equation of type

$$\Lambda_L(s) = \omega \overline{\Lambda_L(1 - \bar{s})},$$

where

$$\Lambda_L(s) = L(s)Q^s \prod_{j=1}^k \Gamma(\lambda_j s + \nu_j)$$

with positive real numbers Q, λ_j and complex numbers ν_j, ω with $Re \nu_j \geq 0$ and $|\omega| = 1$;

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