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ON $|K^\lambda|$ SUMMABILITY OF ORTHOGONAL SERIES

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Abstract. In this paper we have proved two theorems pertaining to $|K^\lambda|$ summability of orthogonal series.

1. Introduction

Let $\{\psi_n(x)\}$ be an orthonormal system defined in the interval (a, b). We assume that $f(x)$ belongs to $L^2(a, b)$ and

$$f(x) \sim \sum_{n=0}^{\infty} c_n \psi_n(x), \tag{1.1}$$

where $c_n = \int_a^b f(x)\psi_n(x)dx$, $(n = 0, 1, 2, \dots)$. By The Riesz–Fischer theorem, for the existence of the function f such that $c_n = \int_a^b f(x)\psi_n(x)dx$ for every n, a necessary and sufficient condition is the convergence of the series $\sum a_n^2$.

Let $\sum_{n=0}^{\infty} a_n$ be a given infinite series with its partial sums s_n and let σ_n^α denotes the nth Cesàro mean of order α (see [4]) such that

$$\sum_{n=1}^{\infty} n^{k-1}|\sigma_n^\alpha - \sigma_{n-1}^\alpha|^k < \infty. \tag{1.2}$$

A series $\sum_{n=0}^{\infty} a_n$ is said to be absolutely summable by Cesàro method of $\alpha > -1$ and index $k \geq 1$, symbolically

$$\sum_{n=0}^{\infty} a_n \in |C, \alpha|, \quad (\alpha > -1, k \geq 1),$$

if (1.2) holds.

Let p_n be a sequence of positive numbers such that

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