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BLOCK SEQUENCES OF RETRO BANACH FRAMES

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Abstract. In this note, we introduce block sequences for a retro Banach frame and exhibit the existence with examples and counter examples. Also, we give a necessary and sufficient condition for a block sequence of a retro Banach frame to be a retro Banach frame. We give a condition (a necessary and sufficient) under which a block sequence of an exact retro Banach frame is an exact retro Banach frame. Finally, we discuss exact retro Banach frames and prove a result related to a geometric property of the underlying space.

1. Introduction and Preliminaries

Throughout H (with inner product $\langle\cdot,\cdot\rangle$) will denote a separable Hilbert space, X a Banach space , X^* denote the conjugate space of X and $Codim_{(X^*)}[f_n]$ is the dimension of quotient spaces of $(X^*/[f_n])$. While working with families of exponentials $\{e^{i\lambda_n t}\}_{n\in\mathbb{Z}}$, Duffin and Schaeffer [6], tried to determine when they are complete or form a Reisz basis for $L^2[a,b]$ etc, and in the process defined frames. They gave the following definition of frame:

A sequence $\{x_n\}$ in a Hilbert space H (with inner product $\langle\cdot,\cdot\rangle$) is said to be a frame (Hilbert) for H, if there exist positive constants A and B such that

$$A\|x\|^2 \leq \sum_n |\langle x, x_n\rangle|^2 \leq B\|x\|^2, \quad \text{for all } \ x \in H \tag{1.1}$$

The positive constants A and B, respectively, are called *lower* and *upper frame bound* for the frame $\{x_n\}$ and collectively known as *frame bounds* for the frame $\{x_n\}$. But they are not unique. In fact, any pair of positive constants A and B for which the inequality (1.1), called *frame inequality* for the frame $\{x_n\}$, hold good are called frame bounds for the frame $\{x_n\}$. The supremum of all lower frame bounds and the infimum of all upper

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