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THE SPECTRAL AND BOUNDEDNESS RADII DEFINING AN EXTREMAL TOPOLOGY

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Abstract. It is characterized when the spectral radius in a Hausdorff locally A-convex algebra is an m-convex norm that defines the weakest m-convex topology stronger than the original one. The same is done for the boundedness radius on A-normed algebras.

1. Introduction

Throughout this paper X will be a complex associative algebra. It is called almost commutative if X is commutative modulo its Jacobson radical. When X is endowed with a topology τ we shall write (X, τ). The linear topology generated by a family $\mathcal{P}$ of seminorms on X will be denoted by $\sigma(\mathcal{P})$. Then $(X, \sigma(\mathcal{P}))$ is a locally convex linear space. When $\mathcal{P}$ consists of only one seminorm $\|\cdot\|$ we simply write $(X, \|\cdot\|)$.

The algebra X with a linear topology τ for which multiplication is separately continuous (respectively, jointly continuous) is called a *semitopological algebra* (respectively, *topological algebra*).

A *locally convex algebra* is a semitopological algebra whose topology is defined by a family of seminorms

The concepts of absorbing seminorm and locally absorbing convex algebra were introduced in [3]. They are called in short form A-convex seminorm and A-convex algebra, respectively. The m-convex seminorms and m-convex algebras are special cases of these concepts. Their definitions of all of them are recalled in the next section.

Every A-convex algebra is a locally convex algebra. While every m-convex algebra is a locally convex algebra with jointly continuous multiplication and therefore, a topological algebra.

When (X, τ) is an A-convex algebra, then there always exists an m-convex topology on X stronger than τ. This fact was proved, for X unital, by M. Oudadess in [12] using the operator topology $Op(\mathcal{P})$, where $\mathcal{P}$ is any family of A-convex seminorms such that

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