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SOME CRITICAL POINT RESULTS FOR FRÉCHET MANIFOLDS

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Abstract. We prove a so-called linking theorem and some of its corollaries, namely a mountain pass theorem and a three critical points theorem for Keller C^1-functional on C^1-Fréchet manifolds. Our approach relies on a deformation result which is not implemented by considering the negative pseudo-gradient flows. Furthermore, for mappings between Fréchet manifolds we provide a set of sufficient conditions in terms of the Palais-Smale condition that indicates when a local diffeomorphism is a global one.

1. Introduction

For Banach and Hiblert manifolds there are two approaches to the critical point theory. One is based on deformation techniques along the negative gradient flow or a suitable substitute of it, namely the pseudo-gradient flow. The other one relies on various versions of Ekeland's variational principle. At the core of both approaches lie Palais-Smale compactness-type conditions. However, as pointed out in [4] these approaches do not work in full extent for more general context of Fréchet manifolds. Since for Fréchet manifolds cotangent bundles do not admit smooth manifold structures and consequently the notion of pseudo-gradient vector fields and Finsler structures on cotangent bundles make no sense.

In this regard, it was introduced the Palais-Smale condition on Fréchet manifolds by using an auxiliary function to detour the need of a smooth structure on cotangent bundles in [4]. The idea behind the definition is that on sets where a real-valued functional on a manifold has no critical points and satisfies the proposed Palais-Smale condition, the associated auxiliary function is negative (Lemma 3.5). In this case, the functional satisfies the hypotheses of the deformation result (Lemma 3.6) which requires that the associated auxiliary function be negative. Moreover, by imposing the closedness

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