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REFINEMENTS OF CARLEMAN'S INEQUALITY

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Abstract. In this paper, we prove that the inequalities

$$\sum_{n=1}^{\infty}(\prod_{k=1}^{n}a_k)^{\frac{1}{n}} \leq e\sum_{n=1}^{\infty}(1-\frac{a}{n+13a})a_n$$

and

$$\sum_{n=1}^{\infty}[(1+\frac{c}{n+b})(\prod_{k=1}^{n}a_k)^{\frac{1}{n}}] \leq e\sum_{n=1}^{\infty}a_n$$

hold if $a_n \geq 0$ $(n = 1, 2, \cdots)$ with $0 < \sum\limits_{n=1}^{\infty} a_n < +\infty$, where $a = 2.739$, $b = [(4\sqrt{2}-3)e-6]/[(3-2\sqrt{2})e] = 2.6203\cdots$ and $c = (e/2-1)(1+b) = 1.3002\cdots$.

1. Introduction

Let $a_n \geq 0$ $(n = 1, 2, \cdots)$ with $0 < \sum\limits_{n=1}^{\infty} a_n < +\infty$, then the well-known Carleman's inequality [1] is given by

$$\sum_{n=1}^{\infty}(\prod_{k=1}^{n}a_k)^{\frac{1}{n}} < e\sum_{n=1}^{\infty}a_n.$$

In the last ninety years, the refinement, improvement, generalization, extension and application for the Carleman's inequality have attracted the attention of many researchers [2-19].

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