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REGULARITY AND CONSTRUCTION OF BOUNDARY MULTIWAVELETS

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Abstract. The conventional way of constructing boundary functions for wavelets on a finite interval is to form linear combinations of boundary-crossing scaling functions. In this article we focus instead on boundary functions defined by recursion relations. We show that the number of boundary functions at each end is uniquely determined, and derive conditions for determining regularity from the boundary recursion coefficients. We then develop an algorithm based on linear algebra which can be used to construct boundary functions with maximal regularity.

1. Introduction

The Discrete Wavelet Transform (DWT) is designed to act on infinitely long signals. For finite signals, the algorithm breaks down near the boundaries. This can be dealt with by extending the data by zero padding, extrapolation, symmetry, or other methods [4, 9, 11, 12], or by constructing special boundary functions [3, 6, 7].

Boundary functions can be constructed as linear combinations of boundary-crossing interior functions, or from boundary recursion relations. The standard examples in the literature have both properties, but the recursion relation approach can produce new boundary functions which cannot be derived using linear combinations. There are also linear combinations which are not refinable, but these are of little practical value, since the DWT algorithm requires recursion coefficients.

One way to find suitable boundary recursion coefficients is based on linear algebra. The infinite banded Toeplitz matrix representing the DWT is replaced by a finite matrix, by suitable end-point modifications.

A particular such method for scalar wavelets is presented in Madych [9]. The Madych approach can be generalized to multiwavelets under an additional assumption, which may or may not be satisfied for a given multiwavelet. We present a modified method which does not require this extra assumption.

Linear algebra completions are not unique; they all include multiplication by an arbitrary orthogonal matrix. A random choice of matrix does not in general produce

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