Poincare Journal of Analysis & Applications
Vol. 2, No. 2 (2015), Special Issue (IWWFA-II, Delhi), 129-137

DOI: 10.46753/pjaa.2015.v02i02.010

ON PERTURBATION OF LOCAL ATOMS FOR SUBSPACES

Deepshikha and L.K. Vashisht†

Date of Receiving	:	02. 05. 2015
Date of Revision	:	29. 07. 2015
Date of Acceptance	:	04. 10. 2015

Abstract. A family of local atoms is a collection of vectors which are analysis and synthesis systems with frame-like properties for closed subspaces of a separable Hilbert space $\mathcal{H}$. In this paper, we present some perturbation results for local atoms in a subspace of a Hilbert space. Some algebraic properties of one of derivatives of local atoms are given.

1. Background

Frames in a Hilbert space are a redundant system of vectors which provides a series representation for each vector in the space. Duffin and Schaeffer [10] in 1952, introduced frames for Hilbert spaces, in the context of nonharmonic Fourier series. Let $\mathcal{H}$ be an infinite dimensional separable real (or complex) Hilbert space with inner product $\langle .,. \rangle$ linear in first entry. A countable sequence $\{f_k\} \subset \mathcal{H}$ is called a *frame* (or *Hilbert frame*) for $\mathcal{H}$ if there exist numbers $0 < m_o \leq M_o < \infty$ such that

$$m_o \|f\|^2 \leq \sum_k |\langle f, f_k \rangle|^2 \leq M_o \|f\|^2 \text{ for all } f \in \mathcal{H}. \tag{1.1}$$

The numbers m_0 and M_0 are called *lower* and *upper frame bounds*, respectively. They are not unique. If it is possible to choose $m_o = M_o$, then the frame $\{f_k\}$ is called *Parseval frame* (or *tight frame*). If the upper inequality in (1.1) satisfied, then we say that $\{f_k\}$ is a *Bessel sequence* in $\mathcal{H}$. Frames were revived by Daubechies, Grossmann and Meyer in [9]. For the utility of frames in different directions in applied mathematics an interested may refer to [2, 6].

Let $\{f_k\}$ be a frame for $\mathcal{H}$. The operator $S : \mathcal{H} \to \mathcal{H}$ given by

$$Sf = \sum_k \langle f, f_k \rangle f_k$$

2010 *Mathematics Subject Classification.* 42C15, 42C30, 42C05, 46B15.
Key words and phrases. Hilbert frame, perturbation, local atoms, K-frames.
The first author is supported by CSIR, India vide File No.: 09/045(1352)/2014-EMR-I.
Lalit K. Vashisht is partly supported by R & D Doctoral Research Programme, University of Delhi, Delhi-110007, India. Grant No. : RC/2014/6820.
Communicated by. Laura Gavruta
†Corresponding author.