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CONSTRUCTIONS OF MRA-BASED WAVELETS AND FRAMES IN WALSH ANALYSIS

Yu. A. Farkov

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Abstract. In this paper, we review the basic constructions of MRA-based orthogonal wavelets and tight frames related to the generalized Walsh functions. Several differences between the constructed wavelets and the classical wavelets are indicated. Further, for any integer $p \geq 2$, all compactly supported step refinable functions on the Vilenkin group G_p are characterized. In addition, we include references to the following topics: biorthogonal wavelets on G_p, approximation properties of compactly supported orthogonal wavelets and frames on the Vilenkin/Cantor groups, unconditional convergence of wavelet expansions, nonstationary, periodic, and discrete wavelets in Walsh analysis, p-multiresolution analysis on the positive half-line, and related subjects.

1. Introduction

Given an integer $p \geq 2$, the Vilenkin group G_p consists of sequences $x = (x_j)$, where $x_j \in \{0, 1, \dots, p-1\}$ for $j \in \mathbb{Z}$ and with at most finite number of negative j such that $x_j \neq 0$. The zero sequence is denoted by θ. If $x \neq \theta$, then there exists a unique $k = k(x)$ such that $x_k \neq 0$ and $x_j = 0$ for all $j < k$. The group operation $\oplus$ on G is defined as the coordinatewise addition modulo p,

$$(z_j) = (x_j) \oplus (y_j) \iff z_j = x_j + y_j \,(\mathrm{mod}\, p) \quad \text{for all} \quad j \in \mathbb{Z};$$

the topology on G_p is introduced via the complete system of neighbourhoods of zero

$$U_l = \{(x_j) \in G_p \,:\, x_j = 0 \text{ for all } j \leq l\}, \quad l \in \mathbb{Z}.$$

The equality $z = x \ominus y$ means that $z \oplus y = x$. For $p = 2$ we have $x \oplus y = x \ominus y$ and the group G_2 coincides with the locally compact Cantor group $\mathcal{C}$.

Notice that for $p = 2$ the subgroup U_0 is isomorphic to the compact Cantor group $\mathcal{C}_0$; i.e., the topological Cartesian product of a countable set of cyclic groups with discrete

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