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# $p$-ADIC WAVELETS

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***Abstract.*** This paper is a short survey on wavelet and MRA theory for functions of $p$-adic argument.

## 1. Introduction

A variety of different orthogonal wavelet bases has been found for $L_2(\mathbb{R})$ for the last three decades. Such bases are not only interesting by themselves, a number of theoretical and applied problems were solved with using them. A general scheme for construction of wavelets of real argument was developed by Meyer and Mallat [19], [20] (see also, [7], [21]). This scheme is based on the concept of multiresolution analysis (MRA in the sequel). The theory allowed to construct wavelet bases and frames with desired properties, in particular, the Daubechies wavelets [7]

It appeared that similar constructions also exist for functions defined on some other algebraic structures, such as the Cantor and Vilenkin groups, local fields of positive characteristic (see, e.g., [17], [22], [10], [4], [5], [6]). Orthogonal wavelet bases essentially different from the Haar basis were found on these structures.

There were several attempts to develop MRA theory with natural translations in a more general setting, e.g., for locally compact groups or zero-dimensional groups (see [3], [18]). Analyzing these papers, one can see that actually nothing except for the Haar basis and its trivial modifications was constructed. $p$-Adic wavelet theory gives an explanation for these failures. From the wavelet theory point of view, the additive group of $p$-adic numbers looks very similar to the Cantor/Vilenkin groups. In both structures, Schwartz-Bruhat functions (i.e., band-limited and compactly supported ones) are the most convenient for MRA constructions. A variety of different orthogonal wavelet bases generated by Schwartz-Bruhat functions has been found for the Cantor/Vilenkin groups in [17], [22], [10]. However the situation is quite different in p-adics. Namely, any

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