# O-FRAMES FOR OPERATORS IN BANACH SPACES

Oleg Reinov

**Abstract**. A new notion of O-frame for an operator $T : X \to Y$ in Banach spaces is introduced: we say that a system $\mathcal{F} := ((x'_k)_{k=1}^\infty, (y_k)_{k=1}^\infty)$ is O-frame (operator frame) for $T$, if for every $x \in X$ the series $\sum_{k=1}^\infty \langle x'_k, x \rangle y_k$ converges in $Y$ and

$$Tx = \sum_{k=1}^\infty \langle x'_k, x \rangle y_k, \quad x \in X.$$

This is an evident, but, as we hope, interesting generalization of the well known notion of a Banach (Schauder) frame. We used the classical tools as well as (and mainly - also now classical) the technique, developed by Aleksander Pełczyński, to get first general properties of O-frames.

We present a small survey on the topic. Let us mention only that, in particular, as in the case of the identity operators, it can be proved that

(i) an operator has an operator frame
if and only if
(ii) it factors through a Banach space with a basis,
and, when it is defined on a separable Banach space, iff
(iii) it has the bounded approximation property.
Also, we consider a new notion of an unconditional operator frame and present some similar results.

## 1. O-frames

We use the standard terminology and notations of the theory of operators in Banach spaces (see, e.g., [6]). All definition, results and details, concerning the notions of frames in Hilbert and Banach spaces, can be found in [1], [2], [5]. Recall only a definition of a Schauder frame for a Banach space (cf. [2] or [5]): a system $(x'_k)_{k=1}^\infty \subset X^*, (x_k)_{k=1}^\infty \subset X$ is said to be a Schauder frame for $X$, if for every $x \in X$ the series $\sum_{k=1}^\infty \langle x'_k, x \rangle x_k$ converges in $X$ to $x$. An evident generalization is given by the following

**Definition 1.1.** Let $T \in L(X, W)$, $(x'_k)_{k=1}^\infty \subset X^*, (w_k)_{k=1}^\infty \subset W$. We say that $\mathcal{F} := ((x'_k)_{k=1}^\infty, (w_k)_{k=1}^\infty)$ is an O-frame (operator frame) for $T$, if for every $x \in X$