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WEIGHTED ABSOLUTE CONVERGENCE OF THE SERIES OF FOURIER-HAAR COEFFICIENTS

Boris Golubov † and Sergey Volosivets

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Abstract. We formulate some sharp statements on absolute convergence of the series of Fourier-Haar coefficients. The two-dimensional analogs of some one-dimensional results are also formulated.

1. Introduction

The Haar orthonormal system $\{\chi_n\}_{n=1}^{\infty}$ had been constructed in 1909 (see [1]). By this system A. Haar gave positive answer on the question of D. Hilbert: is there an orthogonal system such that Fourier seies with respect to this system of any continuous function converges uniformly to that function.

Let us remind the definition of Haar system. We set $\chi_1(x) \equiv 1$ on $[0,1]$. After that we introduce the open dyadic intervals $I_i^k = ((i-1)/2^k, i/2^k)$, $i = 1, 2, \ldots, 2^k$, $k = 0, 1, \ldots,$ and represent the natural number $n \geq 2$ in the form $n = 2^k + i$, $i = 1, 2, \ldots, 2^k$, $k = 0, 1, \ldots$. Then we set $\chi_n(x) = 2^{k/2}$ for $x \in I_{2i-1}^{k+1}$, $\chi_n(x) = -2^{k/2}$ for $x \in I_{2i}^{k+1}$ and $\chi_n(x) = 0 \quad x \in [0,1] \setminus \overline{I_i^k}$, where $\overline{I_i^k}$ is closure of the interval I_i^k. If the Haar function $\chi_n(x)$ has a jump in some point $x \in (0,1)$, then $\chi_n(x) = (\chi_n(x-0) + \chi_n(x+0))/2$. In the endpoints of the interval $[0,1]$ we set $\chi_n(0) = \chi_n(+0)$ and $\chi_n(1) = \chi_n(1-0)$. The Haar functions $\chi_n(x)$ are step functions.

The principal information on Fourier-Haar series may be found in the book [2].

For the function $f \in L_p[0,1]$, $1 \leq p < \infty$, we introduce integral modulus of continuity

$$\omega(\delta, f)_p = \sup_{0 \leq h \leq \delta} \left(\int_0^{1-h} |f(t+h) - f(t)|^p \, dt \right)^{1/p} \qquad (0 \leq \delta \leq 1) \tag{1.1}$$

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†Corresponding author.