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VANISHING MOMENTS OF HILBERT TRANSFORM OF WAVELETS

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Abstract. A wavelet is a localised function having a prescribed number of vanishing moments. Hilbert transform of a wavelet is again a wavelet. In this paper, we discuss the higher vanishing moments of Hilbert transform of wavelets under certain conditions on wavelet and its Hilbert transform.

1. Introduction

Hilbert transform and its basic properties were developed mainly by G.H. Hardy [2, 3] and simultaneously by E.C. Titchmarsh [9, 10]. Hilbert transforms were initially defined for periodic functions. Hilbert transform is a basic tool in Fourier Analysis which provides a definite means for obtaining the harmonic conjugate of a given function or fourier series. It plays an important role in Fluid Mechanics, Aerodynamics, Signal Processing and Electronics. The theory of Hilbert transform also finds a number of applications in pure mathematics. It has been observed in [8] that if $\psi(t)$ is a real wavelet, then Hilbert transform of $\psi(t)$ is also a real wavelet with same energy and admissibility coefficient of its generating wavelet. The higher vanishing moments of wavelets under various conditions on $\psi(x)$ have been investigated in [1, 4, 11].

The aim of this paper is to study the higher vanishing moments of Hilbert transform of wavelets under certain conditions. We begin with the following definition of Hilbert transform of order n given in [6].

For a non-negative integer n, the Hilbert transform of n^{th} order is given by

$$\mathcal{H}^n f(x) = \begin{cases} (-1)^{n/2} f(x), \text{ if } n \text{ is even} \\ (-1)^{(n-1)/2}\mathcal{H} f(x), \text{ if } n \text{ is odd}, \end{cases} \tag{1.1}$$

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