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PERTURBATION OF P-APPROXIMATE SCHAUDER FRAMES FOR SEPARABLE BANACH SPACES

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Abstract. Paley-Wiener theorems for frames for Hilbert spaces, Banach frames, Schauder frames and atomic decompositions for Banach spaces are known. In this paper, we derive Paley-Wiener theorem for p-approximate Schauder frames for separable Banach spaces. We show that our result gives Paley-Wiener theorem for frames for Hilbert spaces.

1. Introduction

About a century old theorem of Paley and Wiener states that sequences which are close to orthonormal bases for Hilbert spaces are Riesz bases (see Chapter 1, Theorem 13 in [21] and [1]). Since frames are generalizations of Riesz bases, we naturally ask whether a sequence which is close to a frame is a frame? Recall that a sequence $\{\tau_n\}_n$ in a separable Hilbert space $\mathcal{H}$ over $\mathbb{K}$ ($\mathbb{R}$ or $\mathbb{C}$) is said to be a frame for $\mathcal{H}$ if there exist $a, b > 0$ such that

$$a\|h\|^2 \leq \sum_{n=1}^{\infty} |\langle h, \tau_n\rangle|^2 \leq b\|h\|^2, \quad \forall h \in \mathcal{H}.$$

Constants a and b are called as lower and upper frame bounds, respectively [12]. First Paley-Wiener theorem (also known as perturbation theorem) of a frame for a Hilbert space is due to Christensen, in 1995, which states as follows.

Theorem 1.1. *[8] Let $\{\tau_n\}_n$ be a frame for $\mathcal{H}$ with bounds a and b. If $\{\omega_n\}_n$ in $\mathcal{H}$ satisfies*

$$c := \sum_{n=1}^{\infty} \|\tau_n - \omega_n\|^2 < a,$$

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