

ON THE SPECTRAL PROPERTIES OF NON- SELF-ADJOINT ELLIPTIC SYSTEMS OF PARTIAL DIFFERENTIAL OPERATORS

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Abstract. Let Ω be a bounded domain in R^n with smooth boundary $\partial\Omega$. In this paper, we will study the spectral properties of degenerate non-selfadjoint elliptic systems of differential operator P acting in Hilbert space $H_\ell = L^2(\Omega)^\ell$ and defined by $(Pu)(x) = -\sum_{i,j=1}^n (\rho^{2\alpha}(x)a_{ij}(x)Q(x)u'_{x_i}(x))'_{x_j}$, with Dirichlet-type boundary conditions, indeed this paper was written in continuing on earlier our papers [10], [14], the paper is sufficiently more general than the paper [10]. Here $c|s|^2 \leq \sum_{i,j=1}^n a_{ij}(x)s_i\overline{s_j}$ ($s = (s_1, \dots, s_n) \in \mathbf{C}^n$, $x \in \Omega$), $\rho(x) = \text{dist}\{x, \partial\Omega\}$, $a_{ij}(x) = \overline{a_{ji}(x)} \in C^2(\overline{\Omega})$, $0 \leq \alpha < 1$. Suppose that $Q(x) \in C^2(\overline{\Omega})$, $End \mathbf{C}^\ell$ such that for each $x \in \overline{\Omega}$ the matrix function $Q(x)$ has non-zero simple eigenvalues $\mu_j(x) \in C^2(\overline{\Omega})$ ($1 \leq j \leq \ell$) arranged in the complex plane such that: $\mu_1(x), \dots, \mu_\ell(x) \in \mathbf{C} \setminus \Phi$, where $\Phi = \{z \in \mathbf{C} : |\arg z| \leq \varphi\}$, $\varphi \in (0, \pi)$.

1. Introduction

Let Ω be a bounded domain in R^n with smooth boundary $\partial\Omega$ (i.e., $\partial\Omega \in C^\infty$). We introduce the weighted Sobolev space $\mathcal{H} = W_{2,\alpha}^2(\Omega)$ as the space of complex value functions $u(x)$ defined on Ω with the finite norm:

$$|u|_+ = \left(\sum_{i=1}^n \int_{\Omega} \rho^{2\alpha}(x) |u'_{x_i}(x)|^2 dx + \int_{\Omega} |u(x)|^2 dx \right)^{1/2}.$$

We denote by $\overset{\circ}{\mathcal{H}}$ the closure of $C_0^\infty(\Omega)$ in \mathcal{H} with respect to the above norm. i.e., $\overset{\circ}{\mathcal{H}}$ is the closure of $C_0^\infty(\Omega)$ in $W_{2,\alpha}^2(\Omega)$. The notion $C_0^\infty(\Omega)$ stands for the space of infinitely differentiable functions with compact support in Ω . In this paper, we investigate the

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