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# REPRESENTATION OF BOUNDED LINEAR OPERATORS ON LAPLACE INTEGRABLE FUNCTIONS

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***Abstract*.** In this paper, a norm is defined on $\mathcal{LP}[a,b]$, the space of all Laplace integrable functions on the closed and bounded interval $[a,b]$, and it is shown that concerning that norm, $\mathcal{LP}[a,b]$ is not complete. And finally, a representation theorem for bounded linear operators on $\mathcal{LP}[a,b]$ is presented.

## 1. Introduction

In 1909, Riesz [29] proved a theorem which states that all bounded linear functionals $A\colon C[0,1] \to \mathbb{R}$ ($C[0,1]$ is the space of all real-valued continuous functions on $[0,1]$ equipped with the supremum norm $\|\cdot\|_\infty$) can be represented by $A[f(x)] = \int_{[0,1]} f(x)\,d\alpha(x)$, where $\alpha$ is a function of bounded variation on $[0,1]$. This theorem is known as the Riesz representation theorem. After this original work of Riesz, many representation theorems are developed on more general spaces; for example, if $1 \leqslant p < \infty$, $\mu$ is a $\sigma$-finite positive measure on $X$ and $T$ is a bounded linear functional on $L^p(\mu)$ (the space of all $f\colon X \to \mathbb{R}$ such that $\int_X |f|^p < \infty$), then there is a unique $g \in L^q(\mu)$ ($q = p/(p-1)$) such that $T(f) = \int_X fg\,d\mu$ for all $f \in L^p(\mu)$ [31], furthermore, if $X$ is a locally compact Hausdorff space, and $T$ is a positive linear functional on $C_c(X)$ (the space of all real-valued continuous functions with compact support), then there is a unique Radon measure $\mu$ on $X$ such that $T(f) = \int_X f\,d\mu$ for all $f \in C_c(X)$ [6]. Although many representation problems have been successfully proved [5, 8], it is still

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