ON NEAR EXACT $G-$BANACH FRAMES

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Abstract. In this paper, near exact $g-$Banach frames are defined and studied. Examples demonstrating the existence of near exact $g-$Banach frames are given. A sufficient condition for a $g-$Banach frame to be a near exact $g-$Banach frame has been obtained. A Krein-Milman-Rutman type stability result for a $g-$Banach frame has been obtained. Finally, we give an application of near exact $g-$Banach frame related to eigenvalue problem.

1. Introduction

Frames for Hilbert spaces were introduced by Duffin and Schaeffer [3] in 1952 as a tool to study some deep problems in non-harmonic Fourier series. Now a days, frames are regarded as an important and integral tool in signal processing, image processing, data compression, signal detection as well as in the study of Besov spaces and Banach spaces etc. For a nice introduction to frames, one may refer [2].

Feichtinger and Gröchenig [4] extended the concept of frames to Banach spaces and introduced the notion of atomic decomposition for Banach spaces. Later, Gröchenig [5] introduced a more general concept for Banach spaces called Banach frame. He gave the following definition of a Banach frame:

Definition 1.1. Let $\mathcal{X}$ be a Banach space and $\mathcal{X}_{d_1}$ be an associated Banach space of scalar-valued sequences indexed by $\mathbb{N}$. Let $\{f_n\} \subset \mathcal{X}^*$ and $S : \mathcal{X}_{d_1} \to \mathcal{X}$ be given. Then, the pair $(\{f_n\}, S)$ is called a Banach frame for $\mathcal{X}$ with respect to $\mathcal{X}_{d_1}$, if

(1) $\{f_n(x)\} \in \mathcal{X}_{d_1}$, for each $x \in \mathcal{X}$.

(2) there exist positive constants A and B with $0 < A \le B < \infty$ such that

$$A\|x\|_{\mathcal{X}} \le \|\{f_n(x)\}\|_{\mathcal{X}_{d_1}} \le B\|x\|_{\mathcal{X}}, \quad x \in \mathcal{X}. \tag{1.1}$$