EXISTENCE AND NONEXISTENCE RESULTS FOR HIGHER ORDER DIFFERENTIAL EQUATIONS WITH NON-HOMOGENEOUS INTEGRAL BOUNDARY CONDITIONS

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Abstract. The goal of this paper is to establish the existence and nonexistence of positive solutions to differential equations of order p

$$w^{(p)}(x) + a(x)f(w(x)) = 0, \quad x \in [0,1],$$

satisfying non-homogeneous integral boundary conditions

$$w^{(i)}(0) = 0, \ i = 0, 1, 2, ..., p-2, \ w^{(r)}(1) - \eta \int_0^1 g(\tau)w^{(r)}(\tau)d\tau = \lambda,$$

where $r \in \{1, 2, \cdots, p-2\}$ but fixed, $p \geq 3$ and $\eta, \lambda \in (0, \infty)$ are parameters, using the Guo–Krasnosel'skii fixed point theorem.

1. Introduction

The theory of differential equations has been used in modeling of physical, biological and medical sciences as well as economics to find optimum investment strategies. In analyzing real world problems, many mathematical models give rise to either initial value problems or boundary value problems. A particular class of problems involving integral boundary conditions arise in thermoelectricity, chemical engineering, plasma physics, and other fields. In these applied settings, only the positive solutions are relevant. To mention a few papers dealing with problems involving integral boundary conditions, see [5, 8, 1, 10, 7] and for the problems involving non-homogeneous boundary conditions, see [2, 11, 13, 9]. The results are now extended to the problem together with non-homogeneous integral boundary conditions.

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