

## APPROXIMATE SOLUTION FOR PROPORTIONAL-DELAY RICCATI DIFFERENTIAL EQUATIONS BY HAAR WAVELET METHOD

BASHARAT HUSSAIN, AFROZ, AND SHAH JAHAN<sup>†</sup>

Date of Receiving : 09. 09. 2021  
Date of Revision : 02. 01. 2022  
Date of Acceptance : 02. 01. 2022

**Abstract.** In this paper, Haar wavelet based numerical scheme is applied to obtain the approximate solution of Riccati differential equations (RDEs) and proportional-delay Riccati differential equations (PDRDEs) of quadratic nature. The basic idea of this algorithm is that it transforms the differential equations into a system of algebraic equations from which approximate numerical solution of the problem is obtained. Several problems are solved and results are compared with Bezier curves method to show the proficiency and simple applicability of the method. We observed that the absolute error is decreasing uniformly with the increase in level of resolution.

### 1. Introduction

The Riccati differential equations come under the class of nonlinear differential equations. These equations are widely studied for numerous problems of contemporary analysis and its applications, and are not easy to solve explicitly, this makes it interesting to investigate the solutions of these equations. Here, we have considered the following Riccati differential equations:

$$y'(t) = q_1(t) + y(t)(q_2(t) + q_3(t)y(t)), t_0 \leq t \leq t_f, y(t_0) = y_0, \quad (1.1)$$

where  $q_1(t), q_2(t)$  and  $q_3(t) (\neq 0)$  are continuous,  $t_0, t_f$  and  $y(t_0)$  are arbitrary constant and  $y(t)$  is unknown function.

Its proportional-delay variant can be written as

$$y'(t) = \psi(t) + by(t) + cy(\alpha t)(d - y(\alpha t)), t_0 \leq t \leq t_f, y(t_0) = y_0, \quad (1.2)$$

where  $c \neq 0, b, d, y_0 \in C$ , and  $\alpha > 0, \psi(t)$  is a continuous and  $\alpha \neq 1$ . When  $0 < \alpha < 1$  (1.2) yields a retarded equation, whereas  $\alpha > 1$  produces advanced equation. It follows from Picard-Lindelöf theorem that the solutions of (1.1) and (1.2) exist and are unique.

---

2010 *Mathematics Subject Classification.* 65M99, 65N35, 65L03, 65N55, 65L05, 65L60.

*Key words and phrases.* Nonlinear Proportional-delay differential equation, Quadratic Riccati differential equation, Multiresolution analysis, Haar wavelet, Approximate solution.

*Communicated by.* S. K. Kaushik