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SOME PICARD TYPE THEOREMS AND CORRESPONDING NORMALITY CRITERIA IN SEVERAL COMPLEX VARIABLES

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Abstract. In this paper, besides a counterexample to Bloch's principle, normality criteria leading to counterexamples to the converse of Bloch's principle in several complex variables are proved. Some Picard-type theorems and their corresponding normality criteria in $\mathbb{C}^n$ are also obtained.

1. Introduction and Auxiliary Results

Let D to be a domain in $\mathbb{C}^n$ and $\mathcal{F}$ to be a family of holomorphic functions in D. $\mathcal{F}$ is said to be normal in D if each sequence in $\mathcal{F}$ contains a subsequence that converges locally uniformly in D. The aim of this paper is to obtain normality criteria leading to counterexamples to the converse of Bloch's principle in several complex variables. Bloch's principle in one complex variable is extensively studied and the reader may refer to [1, 2, 3, 4, 9, 10, 14, 15, 16], in order. Bloch's principle states that a family $\mathcal{F}$ of holomorphic functions of several complex variables in a domain $D \subset \mathbb{C}^n$ satisfying a certain property $\mathbf{P}$ in D is likely to be normal in D if any entire function possessing the property $\mathbf{P}$ in $\mathbb{C}^n$ reduces to a constant. In one complex variable neither Bloch's principle nor its converse holds in general, for example one may refer to [15, 9]. For the construction of counterexamples to the converse, one needs to find a normality criterion with a certain property and then find out a non-constant entire function satisfying this property in $\mathbb{C}^n$; this sounds interesting particularly from the experience of one complex variable case and this is what we have explored in the present paper in several complex variables case too.

We shall denote by $\mathcal{H}(D)$, the class of holomorphic functions $f : D \to \mathbb{C}$, where $D \subset \mathbb{C}^n$ is a domain, and the open unit ball in $\mathbb{C}^n$ shall be denoted by $\mathbb{D}^n := \{z \in \mathbb{C}^n : \|z\| < 1\}$.

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