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## ON Q-ALGEBRAS AND SPECTRAL ALGEBRAS

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**Abstract.** T. Palmer gave the definition of a spectral seminorm and some equivalent statements for the seminormed algebras to be spectral. Some of these statements have been studied in the locally m-convex case and in the locally pseudoconvex case. In this paper we give some analogous results for the locally m-pseudoconvex algebras. Moreover, different proofs for some known results are given.

## 1. Introduction

In 1947 I. Kaplansky introduced in [9] the notion of Q-algebra (more exactly of Q-ring), that is a topological algebra E in which the set Qinv(E) of quasi-invertible elements of E is open. Examples of Q-algebras can be found in [5] and [6]. It is well known that every Banach algebra is also a Q-algebra. The importance of Q-algebras has been given, for example in [7], [8],[10]. Nowadays Q-algebras are involved in modern differential geometry (see [12, Chapter XI]) and play a significant role in PDE's theory, operator theory, and the  $\Psi^*$ -quantization (see [7, p. 73]).

In 1987 V.Mascioni [13] gave several equivalent conditions for a unital complex normed algebra to be a Q-algebra. Later on T. W. Palmer introduced in [14, p. 294] the notion of a *spectral algebra* as an algebra E in which there exists a seminorm (*spectral seminorm*) p such that  $r(x) \leq p(x)$  for each  $x \in E$ , here r(x) denotes the spectral radius of x. He gave there analogous results to V. Mascioni for seminormed algebras and M. Fragoulopoulou in [6, p. 85, Theorem 3.1] for locally m-convex algebras.

Several characterizations of locally m-pseudoconvex Q-algebras are given in the present paper.

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