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DUAL FRAMES ON FINITE DIMENSIONAL QUATERNIONIC HILBERT SPACE

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Abstract. Khokulan et al. [15] introduced frames for finite dimensional quaternionic Hilbert spaces. In this paper, we will study frames for quaternonic Hilbert spaces and discuss different types of dual frames of a given frame in a quaternionic Hilbert space.

1. Introduction

While working on some deep problems in non-harmonic Fourier series, Duffin and Schaeffer [12] indroduced *frames for Hilbert spaces.* According to the Parseval's identity
" If $\{e_n\}_{n\in\mathbb{N}}$ is an orthonormal bases in a Hilbert space $\mathcal{H}$, then

$$\sum_{n=1}^{\infty}|\langle x,e_n\rangle|^2=\|x\|^2,\quad x\in\mathcal{H}."$$

Thus, the idea of frames emerged in order to provide the relaxation to the Parseval's identity into an inequality. This leads us to the following definition:

"A sequence $\{x_n\}_{n\in\mathbb{N}}\subset\mathcal{H}$ is said to be a *frame* for a Hilbert space $\mathcal{H}$ if there exist positive constants A and B such that

$$A\|x\|^2\leq\sum_{n=1}^{\infty}|\langle x,x_n\rangle|^2\leq B\|x\|^2,\quad \text{for all } x\in\mathcal{H}." \tag{1}$$

The positive constants A and B, respectively, are called lower and upper frame bounds for the frame $\{x_n\}_{n\in\mathbb{N}}$. The inequality (1) is called the *frame inequality* for the frame $\{x_n\}_{n\in\mathbb{N}}$. A frame $\{x_n\}_{n\in\mathbb{N}}$ in $\mathcal{H}$ is said to be

- *tight* if it is possible to choose $A=B$.
- *Parseval* if it is a tight frame with $A=B=1$.

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