APPROXIMATIONS IN HÖLDER'S CLASS AND SOLUTION OF BESSEL'S DIFFERENTIAL EQUATIONS BY EXTENDED HAAR WAVELET

SHYAM LAL AND HARISH CHANDRA YADAV[†]

Abstract. In this paper, extended Haar wavelet has been introduced in the interval $[0, \lambda)$, $\lambda > 0$. It reduces to classical Haar wavelet for $\lambda = 1$. The orthonormality of extended Haar wavelets has been discussed. The convergence analysis of an extended Haar wavelet series of a function f belonging to Hölder's classes $H^{\alpha}[0, \lambda)$ & $H_2^{\alpha}[0, \lambda)$ have been studied. Consequently, the approximations of function f belonging to the generalised Hölder's class have been estimated. The solutions of Bessel's differential equation of order zero have been obtained by the extended Haar operational matrix method for $\lambda = 1$ & 2. These solutions for $\lambda = 1$ & 2 are compared with their exact solutions. It is observed that the extended Haar wavelet solutions and their exact solutions are almost the same. This validates the adopted procedure for solutions of Bessel's differential equation by an extended Haar operational matrix. This is a significant achievement in wavelet analysis.

1. Introduction

The approximations of a function f belonging to $H^{\alpha}[0, 1)$, $H_2^{\alpha}[0, 1)$, $H^{\alpha,w}[0, 1)$, have been determined by researchers Lal and Kumar [8], Debnath [7], Meyer [11], Zygmund [14], Lee and Yueh [9], Aznam and Chowdhury [4] and many others. Working in the same direction, recently Ahmedov et al. [1], Baleanu et al. [5], Umer Saeed et al. [13] have developed the estimators of function by Haar wavelet technique. Generally, the Haar wavelet is defined in the interval [0,1). In nature, there are several actual problems which are associated with the interval $[0, \lambda)$, $\lambda > 0$ instead of [0,1). To deal with such a problem, the classical Haar wavelet and Haar operational matrix are not applicable. Hence Haar wavelet and Haar operational matrix are to be defined in the interval $[0, \lambda)$. These are not considered till now. This short-comming in wavelet analysis motivates