

A NOTE ON THE COMPUTABILITY OF HILBERT-SCHMIDT OPERATOR

POONAM MANTRY, S. K. GANDHI, AND RAKSHA SHARMA[†]

Date of Receiving : 18. 10. 2023
Date of Acceptance : 01. 01. 2024

Abstract. In this paper, we study the computability of Paley-Weiner space. Also, a computability result based on the Classical Sampling Theorem is proved. Finally, we discuss the computability of the Hilbert-Schmidt operator.

1. Introduction

The Turing machine related accession to computability in the broad area of analysis is known as computable analysis. Banach and Mazur [1], Grzegorzczuk [9], Kreitz and Weihrauch [11], Lacombe [14], Pour-El and Richards [18], Turing [19], and many others have made significant contributions to this field. For the representation-oriented accession to computable analysis' the standard idea is to represent infinite objects, such as functions, sets or real numbers by infinite strings over an alphabet (which at the very least comprises the symbols 0 and 1). As a result, a surjective function $\delta : \subseteq \Sigma^\omega \rightarrow X$ can be used to represent a set X , where Σ^ω stands for the set of infinite sequences over Σ , and the inclusion symbol signifies the possibility of a partial mapping. Here, the pair (X, δ) is referred to as a represented space.

In case a machine (Turing) is able to compute endlessly long sequences and converts every sequence p on the (input) tape into the appropriate sequence $F(p)$ on the output tape, then the function $F : \subseteq \Sigma^\omega \rightarrow \Sigma^\omega$ is called computable.

We refer Brattka and Dillhage [6] definition for the concept of a computable function between two represented spaces. Also for various other definitions and terminology related to the contents of this paper, one may refer to the references [3, 9, 13, 16, 17]

The admission of evaluation and type conversion is a distinguishing feature of the function space model.

Definition 1.1. Let δ_i be a representation of X_i , $i = 1, 2, 3$. Then

2010 *Mathematics Subject Classification.* 03F60, 46S30.

Key words and phrases. Computable frame, computable Hilbert-Schmidt operator, computable Hilbert space.

Communicated by. Leena Kathuria

[†] *Corresponding author*