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ON PROPERTY (BW1)

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Abstract. In this paper we study property (Bw1) defined in [18]. We establish for a bounded linear operator defined on a Banach space the necessary and sufficient conditions for which property (Bw1) holds. We discuss the property for operators satisfying the single valued extension property (SVEP). We also study the preservation of property under perturbations by finite rank and nilpotent operators. Certain conditions are explored on Hilbert space operators T and S so that $T \oplus S$ obeys property (Bw1).

1. Introduction and Preliminaries

Let $B(X)$ denote the algebra of all bounded linear operators on an infinite-dimensional complex Banach space X. For $T \in B(X)$, we denote by T^*, $\sigma(T)$, $\sigma_{iso}(T)$, $N(T)$ and $R(T)$ the adjoint, the spectrum, the isolated points of $\sigma(T)$, the null space and the range space of T, respectively. Let us denote by $\propto(T)$ the dimension of the kernel $N(T)$ and by $\beta(T)$ the codimension of the range $R(T)$. Recall that the operator $T \in B(X)$ is said to be an upper semi-Fredholm, if the range $R(T)$ of T is closed and $\propto(T) < \infty$, while $T \in B(X)$ is said to be lower semi-Fredholm if $\beta(T) < \infty$.

An operator $T \in B(X)$ is said to be semi-Fredholm if T is either an upper or a lower semi-Fredholm and Fredholm operator if it is both upper and lower semi-Fredholm. If T is semi-Fredholm, then the index of T is defined by

$$ind(T) = \propto(T) - \beta(T).$$

Let $p(T) := asc(T)$ be the ascent of an operator T i.e., the smallest nonnegative integer n such that $N(T^n) = N(T^{n+1})$. If such an integer does not exist we put $asc(T) = \infty$. Analogously, let $q(T) := dsc(T)$ be the descent of an operator T i.e. the smallest non negative integer such that $R(T^n) = R(T^{n+1})$ and if such an integer does not exist we put $dsc(T) = \infty$. It is well known that if $p(T)$ and $q(T)$ are both finite then $p(T) = q(T)$.

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