NORMAL FAMILIES AND ENTIRE FUNCTIONS SHARING ONE VALUE WITH THEIR k-TH DERIVATIVES

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Abstract. In this paper, we use the idea of normal family to investigate the uniqueness problem of entire functions that share one value with their derivatives and obtain a result which improves and generalizes the recent result due to Zhang and Yang [16]. Also we exhibit an example to show that the condition of our result is the best possible.

1. Introduction, definitions and results

In the paper, we assume that the reader is familiar with standard notation and main results of Nevanlinna Theory (see, e.g., [7, 13]). Again we recall that a meromorphic function a is said to be a small function of f if $T(r, a) = S(r, f)$.

Let f and g be two non-constant meromorphic functions and $a \in \mathbb{C}$. If $f(z) - a$ and $g(z) - a$ have the same zeros with the same multiplicities then we say that $f(z)$ and $g(z)$ share a with CM (counting multiplicities) and if we do not consider the multiplicities then we say that $f(z)$ and $g(z)$ share a with IM (ignoring multiplicities). Also if $g(z) - a = 0$ whenever $f(z) - a = 0$, we write $f(z) = a \Rightarrow g(z) = a$. If $f(z) = a \Rightarrow g(z) = a$ and $g(z) = a \Rightarrow f(z) = a$, we then write $f = a \Leftrightarrow g = a$ and we say that f and g share a IM.

We recall that the order $\rho(f)$ of entire function f defined by

$$\rho(f) = \limsup_{r \to \infty} \frac{\log T(r, f)}{\log r} = \limsup_{r \to \infty} \frac{\log \log M(r, f)}{\log r},$$

where $M(r, f) = \max_{|z|=r} |f(z)|$.

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