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$P^{\maltese}$-PROPERTY AND S-CYCLIC MAPPINGS IN GEODESIC SPACES

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Abstract. We investigate the geometric notions of P and $P^{\maltese}$-properties. This way, we prove a best proximity point existence theorem for, the newly introduced, S-cyclic relatively nonexpansive mappings in the setting of geodesic metric spaces. As a consequence, we get both a best proximity point and a fixed point result for cyclic relatively nonexpansive mappings.

1. Introduction

Since the beginning of the current century, best proximity points of cyclic mappings fulfilling certain contractive conditions have been significantly investigated. Let A, B be nonempty subsets of a metric space X, a mapping $T : A \cup B \longrightarrow A \cup B$ is said to be cyclic (*resp.* noncyclic) if $T(A) \subseteq B$ and $T(B) \subseteq A$ (*resp.* $T(A) \subseteq A$ and $T(B) \subseteq B$). If the distance between a point in $A \cup B$ and its image is the least of all distances between points of A and B, then it's called a best proximity point. Studying the existence of such point, the authors of [6] introduced a new class of mappings and proved, under the condition of proximal normal structure, the existence of best proximty point thereof.

Definition 1.1. ([6]) Retaining the same notations, T is said to be relatively nonexpansive if $d(Tx, Ty) \leq d(x, y)$ for all $x \in A$ and $y \in B$.

Theorem 1.2. ([6]) *Let (A, B) be a nonempty, weakly compact convex pair in a Banach space such that (A, B) has proximal normal structure, then all cyclic relatively nonexpansive mappings on $A \cup B$ have a pair of best proximity points.*

Afterwards, the research on the subject took many different paths. Widening up the framework of the study, introducing more general contractive conditions, weaker conditions on the sets and also defining new algorithms converging to the best proximity point.

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