Poincare Journal of Analysis & Applications
Vol. 10, No. 3 (2023), Special issue, 1-18

DOI: 10.46753/pjaa.2023.v010i03.001

GENERALIZED SOBOLEV TYPE SPACES INVOLVING THE WEINSTEIN TRANSFORM

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Date of Receiving : 02. 08. 2023
Date of Acceptance : 21. 09. 2023

Abstract. In this paper, the space $G^{p,s}_{\omega}(\mathbb{R}^{n+1}_+)$ is considered, and many properties, including completeness and inclusion, are discussed by using the theory of the Weinstein transform. It is shown that the space $S_{\omega}(\mathbb{R}^{n+1}_+)$, is dense in space $G^{p,s}_{\omega}(\mathbb{R}^{n+1}_+)$. The generalized Hankel potential $\mathcal{H}^k$ associated with the Weinstein transform is introduced, and its properties are examined. The L^p-space of all such Hankel potential, denoted by $W^{m,p}_{\omega}(\mathbb{R}^{n+1}_+)$, is defined and it is proven that the generalized Hankel potential $\mathcal{H}^t$ is an isometry of $W^{m,p}_{\omega}(\mathbb{R}^{n+1}_+)$ onto $W^{m+t,p}_{\omega}(\mathbb{R}^{n+1}_+)$.

1. Introduction

Sobolev space was introduced by S. L. Sobolev in the 20th century. This space is a useful and powerful tool that is frequently used in nonlinear analysis, differential geometry, physics, and other areas of the mathematical sciences. This theory is useful for solving the problems of partial differential equations. Pahk and Kang [10], extended the concept of Sobolev space to the generalized distribution spaces of Beurling-Björck type [1, 2], and investigated the Sobolev imbedding theorem, the Rellich's compactness theorem and others, exploiting the weight function ω. Pathak and Pandey [12], introduced the Sobolev space of type $G^{p,s}_{\mu}$ and discussed the properties including completeness and inclusion by using the theory of distributional Hankel transform. The aforesaid authors introduced the Hankel potential $\mathcal{H}^s_{\mu}$ and showed that the Hankel potential $\mathcal{H}^s_{\mu}$ is a continuous linear mapping of the Zemanian space $\mathcal{H}_{\mu}$ into itself. Other properties of Hankel potential were also found in this work. Later on, Pathak and Shrestha [13], defind the space of type $G^{p,s}_{\omega,\mu}$ and discussed many results. Pathak [11], considered the generalized Sobolev space $H^{w}_{\omega}(\mathbb{R}^n)$ which is a generalization of the Sobolev space $H^s(\mathbb{R}^n)$ and developed a multiresolution analysis for the generalised Sobolev space.

2010 *Mathematics Subject Classification.* 46F12, 47A30, 35S05, 46E35.
Key words and phrases. Weinstein transform, Hankel Transform, Sobolev space, Pseudo-differential operators.
Communicated by: Nikhil Khanna
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