*Poincare Journal of Analysis & Applications*
*Vol. 10, No. 3 (2023), Special issue, 165-170*

DOI: 10.46753/pjaa.2023.v010i03.010

# ON $\mathcal{H}$-FRAME INDUCED MONOMORPHISMS FOR A REDUCING SUBSPACE $\mathcal{H}$

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| Date of Receiving | : | 03. 04. 2023 |
| Date of Revision | : | 01. 02. 2024 |
| Date of Acceptance | : | 23. 02. 2024 |

***Abstract***. In this paper, we characterize the $\mathcal{H}$-frame induced monomorphism for a normalized tight $\mathcal{H}$-frame wavelet set in a reducing subspace $\mathcal{H}$. This leads to a construction of normalized tight $\mathcal{H}$-frame wavelet sets. Also, we study symmetric normalized tight $\mathcal{H}$-frame wavelet sets as a special case. Further, fixed point sets of these maps are studied. Different examples are considered for illustration.

## 1. Introduction

A function $\psi \in L^2(\mathbb{R})$ is said to be an *orthonormal wavelet* if the system $\{\psi_{j,k}\}_{j,k\in\mathbb{Z}}$, where $\psi_{j,k} \equiv 2^{j/2}\psi(2^j \cdot -k)$, $j,k \in \mathbb{Z}$, forms an orthonormal basis for $L^2(\mathbb{R})$. If $\{\psi_{j,k}\}_{j,k\in\mathbb{Z}}$ forms (tight, normalized tight) frame for $L^2(\mathbb{R})$, then the function $\psi$ is called a (tight, normalized tight) *frame wavelet* for $L^2(\mathbb{R})$. If the modulus of the Fourier transform of a wavelet is a characteristic function on a measurable set $W$ in $\mathbb{R}$, then the set $W$ is called a *wavelet set*. The Lebesgue measure of a wavelet set is $2\pi$. If the characteristic function on a measurable set is modulus of Fourier transform of a normalized tight frame wavelet, then we call the measurable set as a *frame wavelet set*. The Lebesgue measure of a frame wavelet set is less than or equal to $2\pi$. Thus, a wavelet is particularly a normalized tight frame wavelet and a frame wavelet set is a wavelet set if its Lebesgue measure is $2\pi$. A study of frame wavelet sets can be found in Ref. [1, 2, 3, 4, 5, 6] and [12].

Let $\mathcal{H}$ be a closed subspace of $L^2(\mathbb{R})$. We say that $\mathcal{H}$ is a *reducing subspace* if $f(2^m \cdot -l) \in \mathcal{H}$ for all $f \in \mathcal{H}$ and $m,l \in \mathbb{Z}$. A characterization of reducing subspaces is given as follows:

**Theorem 1.1.** [8] *Let $\mathcal{H}$ be a closed subspace of $L^2(\mathbb{R})$. Then $\mathcal{H}$ is a reducing subspace if and only if $\hat{\mathcal{H}} = L^2(\mathbb{R}) \cdot \chi_{\mathbb{E}}$ for some measurable set $\mathbb{E}$ of $\mathbb{R}$ with the property $\mathbb{E} = 2\mathbb{E}$.*

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2010 *Mathematics Subject Classification.* 42C40, 54H25.
*Key words and phrases.* wavelet set, frame wavelet set, wavelet induced isomorphism, frame induced monomorphism, fixed point set.
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