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FRAMES FOR HILBERT SPACES WITH RESPECT TO ℓ^1-SUM OF FINITE-DIMENSIONAL SPACES

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Abstract. The classic definition of a frame by Duffin and Schaeffer is well known and very useful in analysis and its applications. There are many examples of constructing frames based on classical systems of functions. However, among some function systems, for example, for sequences of discretized values of the reproducing kernel of a Hilbert space, Duffin and Schaeffer frames do not exist. Therefore, from the point of view of the construction of series expansions of type $h = \sum_{n=1}^{\infty} c_n f_n$, it turned out to be productive to use the more general concept of a frame in the situation of a Hilbert space and, especially, a Banach space. In this regard, we mention Aldroubi concept of the p-frame and its subsequent generalizations. In this paper we consider frames with respect to the ℓ^1-sum of finite-dimensional spaces, give some examples and study some properties of such frames.

1. Introduction

It is well known that the frame approach is useful in solving the so-called synthesis problem. By the synthesis problem we mean the problem of representing an arbitrary function f in some function space F by a series $f = \sum_{n=1}^{\infty} c_n f_n$ in terms of elements of a function sequence $\{f_n\}_{n=1}^{\infty}$, where the coefficients $\{c_n\}_{n=1}^{\infty}$ belong to some space of number sequence X. We mention that the solution to the synthesis problem uses various modern methods of function theory and functional analysis, such as greedy algorithms (see [13]), orthorecursive expansions (see [10]), and Banach frames (see [5]). We note that the synthesis problem is useful in various practical problems ranging from numerical methods for solving differential equations to signal theory.

The concept of a frame was introduced by Duffin and Schaeffer [7] in 1952.

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