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DYNAMICAL DUAL FRAMES, ANNIHILATING POLYNOMIALS, AND SPECTRAL RADIUS

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Abstract. We use annihilating polynomials to give new proofs and refinements of characterizations of dynamical frames and dynamical dual frames in finite dimensional vector spaces. We use this approach to prove that every redundant finite frame has infinitely many distinct dynamical dual frames that are generated by an operator T with spectral radius $\rho(T) < \Delta$, where $0 < \Delta < 1$ is arbitrary.

1. Introduction

Dynamical sampling [4, 16] is a recent generalization of classical sampling theory [11] that addresses the problem of recovering a signal from spatio-temporal samples of an evolution process. An important motivating difficulty in dynamical sampling is that one might only have access to sub-Nyquist rate spatial samples which must be superresolved using multiple temporal snapshots of a signal under an evolution operator. The seminal work in [3, 4] revealed that many problems on dynamical sampling in both finite and infinite dimensional settings can be reformulated and addressed in terms of frame theory, and this stimulated a burgeoning body of work connecting frames and dynamical sampling, e.g., [1, 2, 5, 6, 8, 10, 13, 14]. This note focuses on dynamical frames and dynamical dual frames in the finite dimensional space $\mathbb{F}^d$, where $\mathbb{F} = \mathbb{R}$ or $\mathbb{F} = \mathbb{C}$.

Given a finite index set $\mathcal{I} \subset \{0, 1, 2, \ldots\}$, a collection of vectors $\{f_n\}_{n\in\mathcal{I}} \subset \mathbb{F}^d$ is a finite frame for $\mathbb{F}^d$ with frame bounds $0 < A \leq B < \infty$ if

$$\forall f \in \mathbb{F}^d, \qquad A\|f\|^2 \leq \sum_{n\in\mathcal{I}} |\langle f, f_n\rangle|^2 \leq B\|f\|^2,$$

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