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# OPERATORS ASSOCIATED WITH GABOR SYSTEMS IN $\mathcal{L}^2(\mathbb{R}, \mathbb{H})$

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***Abstract***. In this paper, we introduce and study Gabor systems for quaternionic Hilbert space $\mathcal{L}^2(\mathbb{R}, \mathbb{H})$ and explicitly define their analysis and synthesis operators with the help of tensor product. We also provide some characterizations for Bessel systems and Gabor frames. Moreover, a necessary and sufficient condition for the boundedness of the synthesis operator with the help of a matrix valued function is also given. Further, we give the Walnut's representation for the Gabor frame operator in quaternionic setting by introducing the correlation functions. Finally, we give a necessary and sufficient condition for the invertibility and boundedness of the Gabor frame operator.

## 1. Introduction

An important task in signal processing is to investigate signals with the help of a set of basic building blocks that includes a fixed function which is shifted in time and frequency. Gabor systems, whose elements are generated by the time-frequency shifts of a fixed function of $\mathcal{L}^2(\mathbb{R})$, serve this purpose[5, 6, 7]. They are basically known for their applications in auditory signal processing, pseudodifferential operator analysis and uncertainity principle.

The generalization of the concept of Gabor systems to the quaternionic Hilbert space $\mathcal{L}^2(\mathbb{R}, \mathbb{H})$ has become an interesting and significant problem mainly due to its importance in signal processing. The non-commutativity of quaternions makes the construction and generalization of the results of Gabor systems to quaternionic Hilbert spaces, a bit difficult. In 2018, Cerejeiras et al. [3] studied Gabor systems for $\mathcal{L}^2(\mathbb{R}^2, \mathbb{H})$ by defining Gabor frame operator using two-sided windowed quaternionic Fourier transform. For detailed information about quaternionic fourier transforms, refer[1, 4]. Recently, Li [12] also studied the quaternionic Gabor frame operator for $\mathcal{L}^2(\mathbb{R}^2, \mathbb{H})$ through two-sided quaternionic Gabor Fourier transform which is almost a similar approach. Their

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