

SOME PROPERTIES OF GENERALIZED FRAMES FOR OPERATORS

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Abstract. In this paper, we study some interesting properties of K - g -frames. We establish a condition on operator K that makes a K - g -frame a g -frame. Also, we obtain condition under which sum of two K - g -frames turns out to be a K - g -frame. Further, a relationship between dual of sum of two K - g -frames and sum of duals of these K - g -frames has been established. We have also obtained conditions on operators U and V such that $\{\Lambda_j U + \Gamma_j V\}_{j \in J}$ becomes a K - g -frame. Next, the notion of $2K$ -frames has been extended to $2K$ - g -frames and it was observed that every $2K$ - g -frame turns out to be a K - g -frame but converse fails to hold. We establish a condition on operator K such that a K - g -frame becomes a $2K$ - g -frame. We then prove a necessary condition under which the sum of a K - g -frame and $2K$ - g -frame turns out to be a K - g -frame. Also, a result that relates the resolution of identity operator and $2K$ - g -frames has been proved. Finally, we prove a characterization for a g -Bessel sequence to be a $2K$ - g -frame.

1. Introduction

Duffin and Schaeffer [11] during their study of the Nonharmonic Fourier series introduced the concept of frames for Hilbert spaces in 1952. The concept of frames was reintroduced in the year 1986, by Daubechies, Grossman and Meyer [6]. As frames form a redundant system, thereby have abundant applications in pure and applied mathematics like filter bank theory, image processing, sigma-delta quantization, capacity of transmission channel, coding theory, data transmission technology to name a few.

The notion of frame theory has been improved and generalized by many authors. The concept of g -Riesz bases and g -frames was coined by Sun [23]. While trying to generalize Banach frames Kaushik *et. al* [18, 19, 20] introduced the concept of fusion Banach frames in Banach spaces and studied various riveting properties of fusion Banach frames. To study the atomic decomposition systems in Hilbert spaces, L. Găvruta [14] introduced

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