INJECTIVITY PROBLEM VIA g-FRAMES

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Abstract. The main issues in quantum detection theory are the injectivity and state estimation problems, which recently linked and partially studied via frame theory on Hilbert spaces. Inspired by some recent researches and works on the quantum detection problem by (discrete) frames, continuous frames and fusion frames, in this note we examine the quantum detection problem for generalized frames (g-frames). In the theory of g-frames, instead of using the frame element itself, we use a family of bounded operators.We characterize injectivity problem by g-frames and with the help of these characterizations, we give some equivalent conditions for the injectivity problems. We answer the injectivity problems both for trace and Hilbert-Schmidt classes of operators.

1. Introduction

One of the problems that is inspired by engineering applications in signal processing and information theory is quantum detection problem which has a variety of applications in optical communications, including the detection of coherent light signals such as radio, radar and laser signals. Quantum detection theory is a reformulation, in quantum mechanical terms, of statistical decision theory. The quantum detection problem can be split as follows: the injectivity problem and the state estimation problem. In recent years the injectivity linked to frame theory, and it is known that the goal of the injectivity problem is to classify frames that are injective with respect to self-adjoint Hilbert-Schmidt class of operators. Recently, the theory of frames has been studied from the perspective of theoretical physics [34] and the quantum detection and injectivity problem studied in the point of view of continuous and fusion frames and in this paper, we consider the Hilbert space g-frame version's of a quantum detection problem.

Let us first recall some backgrounds and basics about frames and the quantum detection problem.

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