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TRANSCENDENTAL ENTIRE SOLUTIONS OF COMPLEX NONLINEAR PARTIAL DIFFERENTIAL DIFFERENCE EQUATIONS IN $\mathbb{C}^2$

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Abstract. In this paper, we explore the existence and forms of transcendental entire solutions of some second order partial differential-difference equations

$$f(z)^2 + 2\,\alpha\, f(z)\left\{L(z,f) + P_{L(f(z))}\right\} + \left\{L(z,f) + P_{L(f(z))}\right\}^2 = e^{g(z)},$$

$$f(z)^2 + 2\,\alpha\, f(z)\left\{a_1 f(z+c) + P_{L(f(z+c))}\right\} + \left\{a_1 f(z+c) + P_{L(f(z+c))}\right\}^2 = e^{g(z)},$$

and

$$P^2_{L(f(z))} + 2\,\alpha\, P_{L(f(z))} L(z,f) + L(z,f)^2 = e^{g(z)},$$

where $g(z)$ is a non-constant polynomial in $\mathbb{C}^2$, $L(z,f)$ and $P_{L(f(z))}$ are introduced in the Definition 2.1 known as first order linear shift operator and second order partial differential operator respectively. Our results are the extensions of some of the recent results due to Xu *et. al* [31], Li-Xu [19]. Moreover, we exhibit a series of examples in support of our results.

1. Introduction and Preliminaries

The well known classical Fermat-type functional equation is of the following form:

$$f(z)^n + g(z)^n = 1 \tag{1.1}$$

over the field of complex numbers $\mathbb{C}$, where n is a positive integer. Gross [6, 8, 7], Baker [1] and Montel [22], independently characterized the existence of entire and meromorphic solutions of (1.1) when $n \geq 2$. For detailed study, we insist the readers to go through [1, 6, 8, 7, 22]. In 2004, when $m = 2$, Yang and Li [32] studied (1.1) with the replacement

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