

## A BANACH BUNDLE APPROACH TO DYNAMICAL PROCESSES ON CHANGING NETWORKS

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**Abstract.** In this paper, we study dynamical processes where the underlying network is allowed to change in time. In order to do so, we use the theory of Banach bundles which allow us to analyze all changes in the dynamics at the same time in a uniform manner.

### 1. Introduction

When modeling dynamical systems it is common that the dynamics interact between certain subsystems. Nowadays, it is important to couple thousands or even millions of sub-systems linked in a network-like manner. The largest and most well-known network is the world wide web. But there are numerous examples of networks, each with its accompanying dynamics, e.g., electric distribution networks, water supply, or road and railway networks. The difficulty of analyzing such dynamics is then due to the diversity of interactions. Much of the diversity derives from interactions, or coupling, of partial differential equations, that act on sub-systems. There are applications for example in control theory, quantum chaos, spectral theory or inverse problems, just to mention a few.

However, in real-life situations it might happen, that the underlying graph structure changes with time. If we think for example about a road network, it might happen that some roads will be closed due to maintenance or some roads are added to prevent traffic congestion. In the abstract framework of graphs, one would then delete or add certain edges. The dynamical processes that are described on the graphs can therefore also change and the question is, whether we are still able to analyze the changed evolution of the system.

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