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ATTRACTORS AND CHAIN RECURRENCE IN NONCOMPACT SPACE FOR SEMIGROUP OF CONTINUOUS MAPS

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Abstract. In [2, 3], Hurley extended Conley's theorem to noncompact spaces. Its significance is that it characterizes the chain recurrent set of a self-map in terms of attractors. Here, we consider an extension of Hurley's result for a semigroup of continuous maps. Earlier, in [7], we had extended Conley's theorem on compact space for a semigroup of continuous maps.

1. Introduction

A *continuous semigroup* is a set of (non-identity) continuous self maps, of a topological space X, closed under the composition. A semigroup G is said to be generated by a family $\{g_\alpha\}_\alpha$ if every element of G can be expressed as compositions of iterations of the elements of $\{g_\alpha\}_\alpha$. We denote this by $G =< g_\alpha >_\alpha$. The space X is assumed to be Hausdorff and first countable.

M. Hurley had shown that the chain recurrent set for a semiflow is the complement of the union of the set $B(A) \setminus A$, as A varies over the collection of attractors and $B(A)$ denotes the basin of attraction. This concept for flows on a compact space was originally introduced by Conley [1]. Later, in [4] Hurley extended this characterization for semiflows without the assumption of compactness. This paper aims to study the notion of attractors and chain recurrence for noncompact spaces in the context of a continuous semigroup such that the classical case turns out as a special case of this generalization. In [6, 7], we have studied the chain recurrent set and established the notion of attractors for a semigroup of continuous maps. Also, we extended the characterization of the chain recurrent set in terms of attractors on compact metric spaces. Here, we shall follow the treatment of Hurley for noncompact metric spaces and

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