Poincare Journal of Analysis & Applications
Vol. 11, No. 1 (2024), 95-105
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DOI: 10.46753/pjaa.2024.v011i01.007
Online Published on 01. 07. 2024

A NOTE ON RIESZ BASES IN THE FRAMEWORK OF QUATERNIONIC HILBERT SPACE

S. K. SHARMA, VIRENDER† AND S. K. KAUSHIK

Date of Receiving	:	18. 08. 2023
Date of Revision	:	19. 12. 2023
Date of Acceptance	:	29. 04. 2024

Abstract. In this article, we introduce and study Riesz bases in a separable quaternionic Hilbert space. It is proved that a Riesz basis is a frame in the quaternionic Hilbert space. Riesz sequences are defined and equivalence of a Riesz basis and a complete Riesz sequence in a separable quaternionic Hilbert space is proved.

1. **Introduction**

Frames for Hilbert spaces, which plays an important role in many applications, were introduced way back in 1952 by Duffin and Schaeffer [9] as a tool to study of non-harmonic Fourier series. Duffin and Schaeffer introduced frames for particular Hilbert spaces of the form $L^2[a,b]$. They defined a frame as

"A sequence $\{x_n\}_{n\in\mathbb{N}}$ in a Hilbert space $\mathcal{H}$ is said to be a *frame* for $\mathcal{H}$ if there exist constants A and B with $0 < A \le B < \infty$ such that

$$A\|x\|^2 \le \sum_{n=1}^{\infty} |\langle x, x_n\rangle|^2 \le B\|x\|^2, \quad \text{for all } x \in \mathcal{H}." \tag{1.1}$$

Moreover, the positive constants A and B, respectively, are called *lower frame bound* and*upper frame bound*, respectively, for the frame $\{x_n\}_{n\in\mathbb{N}}$. Collectively, these are referred as *frame bounds* for the frame $\{x_n\}_{n\in\mathbb{N}}$. The inequality (1.1) is called the *frame inequality* for the frame $\{x_n\}_{n\in\mathbb{N}}$. A sequence $\{x_n\}_{n\in\mathbb{N}} \subset \mathcal{H}$ is called a *Bessel sequence* if it satisfies upper frame inequality in (1.1) i.e. it has upper bound which satisfies the inequality. A frame $\{x_n\}_{n\in\mathbb{N}}$ in $\mathcal{H}$ is said to be

- *tight* if it is possible to choose $A,\ B$ with $A = B$ satisfying inequality (1.1).

2010 *Mathematics Subject Classification.* 42C15, 42A38.
Key words and phrases. Frames, Quaternionic Hilbert space.
Communicated by: A. Askari Hemmat
† *Corresponding author*