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INTERPOLATION OF ENTIRE FUNCTIONS FROM LATTICE SAMPLING

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Abstract. We extend an interpolation formula for entire functions of order at most 2, minimal type, of J. M. Whittaker from the square lattice $\mathbb{Z}+\mathbb{Z}i$ in $\mathbb{C}$ to more general lattices $\mathbb{Z}\pi+\mathbb{Z}\tau\pi$ where $\Im m\,\tau>0$. With it, we prove that an entire function of order at most 2, minimal type, which is bounded on a lattice in $\mathbb{C}$ is necessarily constant, thus generalizing a result of Pólya. We also derive identities about double series.

1. Introduction

Well-known for its relevance in signal processing, Shannon-Kotel'nikov interpolation theorem states that if a function f in one real variable is band-limited to $[-\pi;\pi]$, that is, if f is the Fourier transform of a function with support in $[-\pi;\pi]$, then it is completely determined by its values at the integers — in formula:

$$f(t)=\sin\pi t\sum_{n=-\infty}^{+\infty}\frac{f(n)}{(t-n)}\frac{1}{(-1)^n\pi} \tag{1.1}$$

(see [2] for the history of this result).

In [11, L_{501} p.72] J. M. Whittaker obtained the following two dimensional generalization of (1.1):

$$f(z)\;=\;\sigma(z)\sum_{(m,n)\in\mathbb{Z}^2}\frac{f(m+in)}{(z-m-in)}\frac{1}{\sigma'(m+in)} \tag{1.2}$$

for all entire functions f such that $\overline{\lim_{r\to\infty}}\log(\max_{|z|=r}|f(z)|)/r^2<\pi/2$, where σ is the Weierstrass sigma function associated to the lattice $\mathbb{Z}+\mathbb{Z}i$ (a definition of σ is given

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