

THE SHARP BOUND OF THE GENERALIZED ZALCMAN FOR THE INITIAL COEFFICIENT AND CERTAIN SECOND HANKEL DETERMINANTS OF k^{th} -ROOT TRANSFORMATION FOR A SUBCLASS OF HOLOMORPHIC FUNCTIONS

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Abstract. The objective of this paper is to estimate the sharp bound of the second Hankel determinants $H_{2,1,k}(f)$, $H_{2,2,k}(f)$ and generalized Zalcman for the initial coefficient for k^{th} -root transformation to the subclass of Holomorphic functions.

1. Introduction

Let \mathcal{A} be the family of mappings f of the type

$$f(z) = z + \sum_{t=2}^{\infty} a_t z^t \quad (1.1)$$

satisfying the normalized conditions $f(0) = 0$ and $f'(0) = 1$ in the open unit disk $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ and \mathcal{S} is the subfamily of \mathcal{A} , possessing univalent (schlicht) mappings.

Let k be a positive integer. A domain $\mathbb{U} \in \mathbb{C}$ is said to be k -fold symmetric if a rotation of \mathbb{U} about the origin through an angle $\frac{2\pi}{k}$ carries \mathbb{U} to itself. A function f is said to be k -fold symmetric in \mathbb{D} if $f\left(e^{\frac{2\pi i}{k}} z\right) = e^{\frac{2\pi i}{k}} f(z)$ for every $z \in \mathbb{D}$. If f is regular and k -fold symmetric in \mathbb{D} , then

$$f(z) = b_1 z + b_{k+1} z^{k+1} + b_{2k+1} z^{2k+1} + \dots \quad (1.2)$$

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