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SOME REMARKS ON RETRO BANACH FRAMES

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Abstract. In this paper, we study exact retro Banach frames and give a sufficient condition for its existence. Also, we prove that if Y is a subspace of infinite co-dimensions of a separable Banach space $\mathcal{B}$ and if $(\mathcal{B}/Y)^*$ has a retro Banach frame, then $\mathcal{B}^*$ has a retro Banach frame. Finally, we define retro Banach frame of type P and proved that a separable Banach space has a retro Banach frame of type P.

1. Introduction

Duffin and Schaeffer [8] developed the notion of frame in the context of Hilbert spaces in 1952. In fact, Duffin and Schaeffer borrowed the Hilbert frame definition technique from Gabor's work [11]. Many years later, in 1986, Daubechies, Grossmann, and Meyer [9] recognized the potential of frames and discovered new uses for them in wavelets, Gabor transforms, and other pure and applied mathematics. For example, strong instruments from operator theory and Banach spaces are being used to investigate frames [12, 14, 18, 19, 20, 21, 23, 28, 31]. The seminal publications by O. Christensen [6, 7] go into further information about the use of frames. Following the publication by Daubechies, Grossmann, and Meyer [9], the idea of frames attracted a lot of attention.

A frame in the Hilbert space $\mathcal{H}$ is a sequence $\{h_i\}_{i=1}^{\infty} \subset \mathcal{H}$ of vectors that fulfills

$$M\|l\|^2 \leq \sum_{i=1}^{\infty} |\langle l, h_i\rangle|^2 \leq K\|l\|^2, \quad \forall\ l \in \mathcal{H},$$

where $0 < M \leq K < \infty$. M is called the lower frame bound and K is called the upper frame bound of the frame $\{h_i\}_{i=1}^{\infty}$ respectively. These frame bounds are not unique. If $M = K$, then the frame $\{h_i\}_{i=1}^{\infty}$ is said to be a tight frame and if $M = K = 1$, then $\{h_i\}_{i=1}^{\infty}$ is called Parseval frame. The above inequality is called the frame inequality.

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