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SPECTRAL CUT-OFF REGULARIZATIONS FOR AN ILL-POSED NON-LINEAR ABSTRACT CAUCHY PROBLEM

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Abstract. We focus on nonlinear abstract Cauchy problem $u_t(t) + Au(t) = f(t, u(t))$, $0 \le t < \tau$ with $u(\tau) = \phi$, where A is a densely defined positive self-adjoint unbounded operator acting on a Hilbert space. This problem is ill-posed in the sense that minor variations in the final value can lead to a significant deviations in the solution. Solution of the considered problem satisfies ill-posed integral equation. Spectral cut-off is used to obtain regularized approximations for the solution of the integral equation. Under the assumptions on the solution $u(\cdot)$ instated of conditions on the final value ϕ and the function $f(\cdot, u(\cdot))$ as in Jana and Nair (2019), we obtain error estimates for exact as well as noisy final value ϕ. Also we investigate stability analysis under appropriate parameter choice strategies. This work extends the work of Nam (2010) from the operator A having discrete spectrum to continuous spectrum. Estimates obtained in this work include the estimates obtained in Tuan (2010), Jana and Nair (2016, 2019) together with various other results present in the existing literature.

1. Introduction

Let H be a Hilbert space over real or complex field, and let $A : D(A) \subset H \to H$ be a densely defined positive self-adjoint unbounded operator. For $\tau > 0$ and $\phi \in H$, consider the problem of solving the *nonlinear Cauchy final value problem*, denoted briefly as nonlinear FVP,

$$u_t(t) + Au(t) = f(t, u(t)), \qquad 0 \le t < \tau \tag{1.1}$$

$$u(\tau) = \phi, \tag{1.2}$$

where $f : [0, \tau] \times H \to H$ is a Borel measurable function such that for any continuous function $v : [0, \tau] \to H$, the function $s \mapsto f(s, v(s))$ belongs to $L^1([0, \tau]; H)$. It is well known that the above problem is ill-posed in the sense that a solution may not exist and if a solution exists, then small perturbations in ϕ can lead to large deviations in

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