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ON MAZUR PROPERTY IN BANACH SPACES WITH PRI

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Abstract. Mazur property is proved in some Banach spaces having projectional resolutions of identity (PRI). As a consequence these Banach spaces (having PRI) have Pettis integration property.

1. Introduction and Preliminaries

The concept of projectional resolutions of the identity (in short, PRI) has played an important tool in the study of geometric properties of Banach spaces like Gelfand-Philips property, having weak* sequentially compact and weak* angelic dual unit balls, existance of Markusévic bases and Odell-Rosenthal type results. In this paper, we shall prove another important property namely Mazur property in some Banach spaces having PRI. As a consequence, these spaces have Pettis Integration property.

An infinite dimensional Banach space is denoted by E and let E^* and E^{**} be its first and second dual respectively. The cardinality of the smallest dense set in E is denoted by dens E. The first ordinal with cardinality $\aleph_0$ is denoted by ω; the other symbols used for ordinals are α, β, λ and μ. Let the first ordinal with cardinality dens E is denoted by μ. A long sequence $\{p_\alpha\}_{\omega\le\alpha\le\mu}$ of projections on a Banach space E is said to be a projectional resolutions of the identity (PRI) of E, if the following conditions are satisfied:

$$(i) \quad \|p_\alpha\| = 1 \qquad (\omega \le \alpha \le \mu)$$

$$(ii) \quad p_\alpha \cdot p_\beta = p_\beta \cdot p_\alpha = p_\beta \qquad (\omega \le \beta \le \alpha \le \mu)$$

$$(iii) \quad \text{dens } p_\alpha(E) \le \text{card } \alpha \qquad (\omega \le \alpha \le \mu)$$

$$(iv) \quad p_\alpha(E) = \overline{\cup\{p_{\beta+1}(E) : \omega \le \beta < \alpha\}} \qquad (\omega \le \alpha \le \mu)$$

$$(v) \quad p_\mu = I.$$

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