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NORMALITY CRITERIONS ON SHARING A HOLOMORPHIC FUNCTION

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Abstract. Let $\mathcal{F}$ be a family of holomorphic functions in a domain D, whose zeros have multiplicity at least 4 in D. Let $\psi(\not\equiv 0)$ be a holomorphic function in D, whose zeros have multiplicity at most 2. If for each pair of functions $\{f, g\} \subset \mathcal{F}$, f' and g' share ψ in D, then $\mathcal{F}$ is normal in D.

1. Introduction and main result

Let $\mathbb{C}$ be the complex plane and D be a domain on $\mathbb{C}$. For $z_0 \in \mathbb{C}$ and $r > 0$, we write $\Delta(z_0, r) := \{z| \, |z - z_0| < r\}$, $\Delta := \Delta(0, 1)$ and $\Delta'(z_0, r) := \{z| \, 0 < |z - z_0| < r\}$. We write $f_n \overset{\chi}{\Rightarrow} f$ in D to indicate that the sequence $\{f_n\}$ converges to f in the spherical metric uniformly on compact subsets of D and $f_n \Rightarrow f$ in D if the convergence is in the Euclidean metric. We also write $\mathbb{P}_f := f^{-1}\{\infty\}$ for the set of poles of a meromorphic function f.

Let f and g be two meromorphic functions and φ be a holomorphic function in D. We say that f and g share φ in D if $\{z \,|\, f(z) = \varphi(z), z \in D\} = \{z \,|\, g(z) = \varphi(z), z \in D\}$.

Our point of departure is the following criterions for normality due to Xu [1].

Theorem 1. [1, Lemma 7] *Let $\mathcal{F}$ be a family of holomorphic functions in D, and let $\psi(\not\equiv 0)$ be a holomorphic function in D. Suppose that, for each $f \in \mathcal{F}$, f has no zeros in D. If for each pair of functions $\{f, g\} \subset \mathcal{F}$, f' and g' share ψ in D, then $\mathcal{F}$ is normal in D.*

Theorem 2. [1, Theorem 1.1] *Let $\mathcal{F}$ be a family of meromorphic functions in D, and let $\psi(\not\equiv 0)$ be a holomorphic function in D whose zeros are simple. Suppose that, for each $f \in \mathcal{F}$, f has no zeros and has only multiple poles in D. If for each pair of functions $\{f, g\} \subset \mathcal{F}$, f' and g' share ψ in D, then $\mathcal{F}$ is normal in D.*

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