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NUMERICAL SOLUTION OF NON-LINEAR LIÉNARD EQUATION USING HAAR WAVELET METHOD

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Abstract. In this article, we propose the Haar wavelet method (HWM) for the numerical solution of linear and non-linear Liénard equations. This method transforms the Liénard equation into a system of algebraic equations using truncated Haar wavelet expansions, which is then solved using Newton's iteration method. Numerical experiments demonstrate the validity and accuracy of HWM, particularly when compared to other existing methods. The desired accuracy for solutions to non-linear Liénard equations is achieved with only a small number of basis points, highlighting HWM as an effective tool for solving both linear and non-linear Liénard equations.

1. Introduction

The real world problems, especially, nonlinear problems, which arise in many physical and scientific fields such as solid-state physics, chemical kinetics, plasma physics, fluid mechanics, and mathematical biology [37, 38], are modelled with nonlinear differential equations [19, 20]. These differential equations, in general, are non-solvable in nature or very hard to achieve the solution. The researchers use various techniques to get the approximate solution of these nonlinear equations. The Liénard equation is one of the differential equations of second order, which is named after the French physicist Alfred-Marie Liénard, in 1928. The general form of the Liénard equation is:

$$y''(x) + f(y)y'(x) + g(y) = b(x) \tag{1.1}$$

where $f(y)$, $g(y)$ are functions of y and $b(x)$ is a function of x.
The equation (1.1) is actually generalization of the damped pendulum equation, where the term $f(y)y'(x)$ is known as the damping force and $f(y)$ is known as the restoring force. It is a special kind of differential equation in mathematics as well as in natural

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