

A NOTE ON ITERATED pg -FRAMES IN BANACH SPACES

VARINDER KUMAR[†], A. ZOTHANSANGA, AND RAKESH KUMAR

Date of Receiving : 14. 11. 2024
Date of Revision : 23. 12. 2024
Date of Acceptance : 24. 12. 2024

Abstract. For a bounded operator Λ on a Banach space X , we propound the structure of the pg -frames generated by iterations of Λ and christen it as iterated pg -frame. We derive sufficient condition on the operator Λ for $\{\Lambda^i\}_{i \in \mathbb{N}}$ to be an iterated pg -frame. Further, we obtain some properties of the operator Λ in case $\{\Lambda^i\}_{i \in \mathbb{N}}$ is an iterated pg -frame for X . Towards the end, we analyze iterated pg -frames of the form $\{\psi\Lambda^i\}_{i \in \mathbb{N}}$ and $\{\Lambda^i\psi\}_{i \in \mathbb{N}}$, where $\psi \in B(X)$.

1. Introduction

Frames in Hilbert spaces is a permeance of a basis of a vector space to the sets which are not necessarily linearly independent. Frames can be visualized as the redundant bases which are generalization of orthonormal bases. The concept of Hilbert space frames were first propounded by Duffin and Schaeffer [10] to study some profound problems in non-harmonic Fourier series. Throughout this paper I denotes a subset of \mathbb{N} , H denotes a seperable Hilbert space and X denotes a Banach space.

Definition 1.1. [10] Let $I \subseteq \mathbb{N}$. A family of vectors $\{x_i\}_{i \in I}$ in a Hilbert space H is said to be a frame for H , if there exist two constants $A, B > 0$ such that

$$A\|x\|^2 \leq \sum_{i \in I} |\langle x, x_i \rangle|^2 \leq B\|x\|^2, \text{ for all } x \in H.$$

The concept of p -frame for Banach spaces was introduced by Christensen and Stoeva [9] and p -frame of subspaces by Faroughi and Najati [15]. Frames have massive fruitful applications in quantum mechanics [4, 19] and Dynamical sampling [2, 3]. Elementary theory of frames and their applications in different directions are categorically explained in the books of Casazza and Kutyniok [4], Christensen [6, 7, 8], Han et.al.[11], Okoudjou [16], the tutorials of Casazza [5] and the memoir of Han and Larson [12]. Over past

2010 *Mathematics Subject Classification.* 42C15, 42A38.

Key words and phrases. frames, g -frames, pg -frames, spectral radius.

Communicated by: Mayur Puri Goswami

[†] *Corresponding author*