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UNIQUENESS AND NON-UNIQUENESS OF THE RADON TRANSFORM

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Abstract. Let $Rh = 0$, R is the Radon transform of h, $Rh = \int_{\mathbb{R}^2} \delta(p-\alpha\cdot x)h(x)dx = \int_{L_{\alpha\, p}} h(s)ds$, where $L_{\alpha\, p}$ is the straight line $\alpha \cdot x = p$, $\alpha = (\cos\theta, \sin\theta)$, $0 \le \theta < 2\pi$. Uniqueness of R means that equation $Rh = 0$ implies $h = 0$. Non-uniqueness means that there exists h, not equal to zero identically and satisfying equation $Rh = 0$ for all unit vectors α and all $p \ge 0$. We prove uniqueness of the Radon transform for $h \in S'$, where S' is the Schwartz's space of tempered distributions. It is known that there are entire functions h, not equal to zero identically, $h = h(z)$, $z = x_1 + ix_2$, which satisfy equation $Rh = 0$.

1. Introduction

The Radon transform of h is defined as

$$Rh = \int_{\mathbb{R}^2} \delta(p - \alpha \cdot x)h(x)dx = \int_{L_{\alpha\, p}} h(s)ds, \tag{1.1}$$

where $L_{\alpha\, p}$ is the straight line $\alpha \cdot x = p$, ds is the element of the length along this straight line, δ stands for the delta-function, $\alpha = (\cos\theta, \sin\theta)$, $0 \le \theta < 2\pi$, and h stands for a locally continuous function, $0 \le p < \infty$. So, the Radon transform depends on p and α.

Uniqueness of R means that equation $Rh = 0$ implies $h = 0$. Non-uniqueness means that there exists h not identically equal to zero and satisfying equation $Rh = 0$ for all unit vectors α and all $p \ge 0$.

In this paper we solve one of the open problems from [7]:

We prove uniqueness of the Radon transform for $h \in S'$, where S' is the Schwartz's space of tempered distributions.

It is known that there are entire functions $h \not\equiv 0$, $h = h(z)$, $z = x_1 + ix_2$, which satisfy equation $Rh = 0$, [4], p. 55, see also [1]. It is known (see [4]) that if $h \in S$ and $Rh = 0$, then $h = 0$.

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