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SYNONYMS OF LOCAL FUNCTION

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Abstract. Through this paper, it will be shown that the study of ideals in a topological space may be replaced by the study of primals, grills, filters etc. in the topological space. To do this, the relationship between local functions via different mathematical structures will be considered. Induced topologies via ideal, primal, grill etc. will also be played an important role for discussing the synonym studies.

1. Introduction

Primal [2, 3, 4], grill [8], ideal [25] and many other mathematical structures can be used to moderate the various types of limit points [2, 19, 21, 25, 34], different types of connectedness [12, 29, 30, 35, 36], several types of compactness [12, 15, 17, 18, 20], distinct types of continuities [1, 27, 28, 32]. These mathematical structures can be expressed in a common frame. Let M be a nonempty set. According to Kuratowski [25], a collection $\mathcal{I} \subseteq 2^M$ is called an ideal on M if $\mathcal{I}$ is closed under hereditary property and finite additivity property. According to Choquet [8], a collection Ω of subsets of a nonempty set M is said to be a grill on M if it satisfies: (i) $\varnothing \notin \Omega$, (ii) $A \in \Omega$ and $A \subseteq B$ implies $B \in \Omega$ and (iii) $A \notin \Omega$ and $B \notin \Omega$ implies $A \cup B \notin \Omega$. According to Acharjee et. al. [2], a collection $\mho$ of subsets of a nonempty set M is said to be a primal on M if it satisfies: (i) $M \notin \mho$, (ii) $A \in \mho$ and $B \subseteq A$ implies $B \in \mho$ and (iii) $A \cap B \in \mho$ implies $A \in \mho$ or $B \in \mho$. Equivalently, a collection $\mho$ of subsets of a nonempty set M is said to be a primal on M if it satisfies: (i) $M \notin \mho$, (ii) $B \notin \mho$ and $B \subseteq A$ implies $A \notin \mho$ and (iii) $A \notin \mho$ and $B \notin \mho$ implies $A \cap B \notin \mho$. Due to the ideal $\mathcal{I}$ on a topological space $(M, \mathcal{T})$, Kuratowski's local function is, $A^* = \{x \in M |\ U_x \cap A \notin \mathcal{I}\ \forall\ U_x \in \mathcal{T}(x)\}$, where $\mathcal{T}(x) = \{U \in \mathcal{T} |\ \ x \in U\}$. It is represented as various types of limit points. Due to this generalization Modak et. al discussed minimal open sets in [31]. But the Corollary 3.18 of [31] will be true only when the space is Hayasi-Samuel [9]. In [21], Janković and Hamlett defined the map $Cl^*_{\mathcal{I}} : 2^M \to 2^M$, by $Cl^*_{\mathcal{I}}(A) = A \cup A^*$ which is a Kuratowski's

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