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DYNAMICS OF HOUSEHOLDER'S METHOD APPLIED TO SMALL DEGREE POLYNOMIALS

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Abstract. In this work, we study the dynamics of Householder's method $h_{f,d}(z)$ of order $d+1$, specially for $d=3$, applied to some small degree polynomials. First, we prove the Scaling theorem and give some remarks on it. We study the Fatou set of $h_{f,d}(z)$ for any quadratic polynomial. The complete dynamics of $h_{f,3}(z)$ for a quadratic unicritical polynomial is studied here. We show that for every polynomial f, the symmetry group $\Sigma(f) = \Sigma(h_{f,d})$ does not always hold for all $d \geq 1$. Also, we study the dynamics of $h_{f,3}(z)$ for any cubic polynomial. Specially, the Fatou set of $h_{f,3}(z)$ for cubic unicritical polynomials contains three unbounded attracting domains. Finally, we study the symmetry and dynamics of $h_{f,3}(z)$ for $f(z) = z(z^2-1)$. Here, we prove that the Fatou set consists of three unbounded attracting basins corresponding to the roots of the function $f(z)$ and the symmetry group $\Sigma(h_{f,3})$ does not contain any translation.

1. Introduction

Let $f : \widehat{\mathbb{C}} \to \widehat{\mathbb{C}}$ be a rational function, where $\mathbb{C}$ is the set of complex numbers and $\widehat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$. A rational function f is of the form $f(z) = \frac{P(z)}{Q(z)}$, where P and Q are polynomials having no common factor. The degree of rational function f is defined as $deg(f)$=max$\{deg(P), deg(Q)\}$. Now, the n^{th} iteration of f is f^n, defined as $f^n(z) = f^{n-1}(f(z))$, with $f^0(z) = z$ being the identity function.

The main motive of the iteration theory is to divide the Riemann sphere into two completely invariant subsets, namely the Fatou set and the Julia set. The Fatou and Julia sets of the function f are denoted by $F(f)$ and $J(f)$, respectively. The Fatou set is defined by $F(f) = \{z \in \widehat{\mathbb{C}} : \{f^n\}$ is normal in some neighbourhood of z for all

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