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APPROXIMATION PROPERTIES OF THE REPEATED DE LA VALLÉE POUSSIN MEANS

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Abstract. We present certain properties of the repeated de la Vallée Poussin means that demonstrate their approximation capabilities. The repeated de la Vallée Poussin means are described in terms of the summability factors of Fourier series. Using these means, we establish the order of approximation for functions belonging to classes of Poisson integrals.

1. Introduction

Let $\mathrm{L}(\mathbb{T})$, where $\mathbb{T}=[-\pi,\pi]$, be the space of summable 2π-periodic functions f. The Fourier series of the function f is given by

$$S[f]=\frac{a_0[f]}{2}+\sum_{k=1}^{\infty}(a_k[f]\cos kx+b_k[f]\sin kx):=\frac{a_0[f]}{2}+\sum_{k=1}^{\infty}A_k[f](x),$$

where

$$a_k[f]=\frac{1}{\pi}\int\limits_{\mathbb{T}}f(x)\cos kx\,dx,\quad b_k[f]=\frac{1}{\pi}\int\limits_{\mathbb{T}}f(x)\sin kx\,dx,\quad k\in\mathbb{Z}_+$$

are the Fourier coefficients of the function f. The n-th partial sum of the Fourier series of the function f is denoted by

$$S_n[f](x)=\frac{a_0[f]}{2}+\sum_{k=1}^{n}A_k[f](x).$$

The Fourier series $S[f]$ of a function $f\in\mathrm{L}(\mathbb{T})$ is said to be summable by the Fejer summation method to a function S if

$$\lim_{n\to\infty}\sigma_n[f](x)=S(x),$$

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